Interpreting the Past: Modeling the 100,000 year Problem

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February 4, 2014

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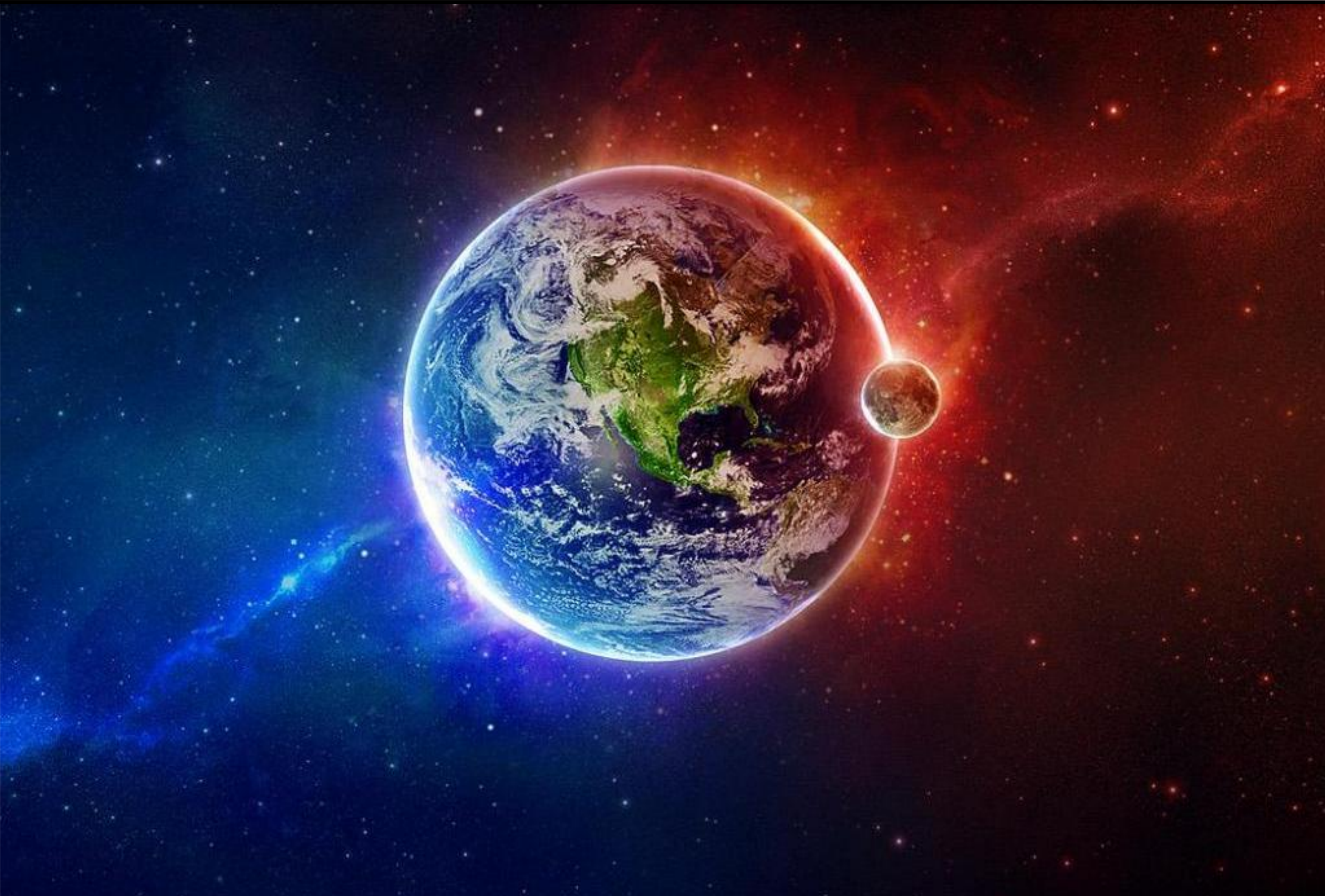
University of Minnesota

Introduction

We are living in our only Petri Dish.

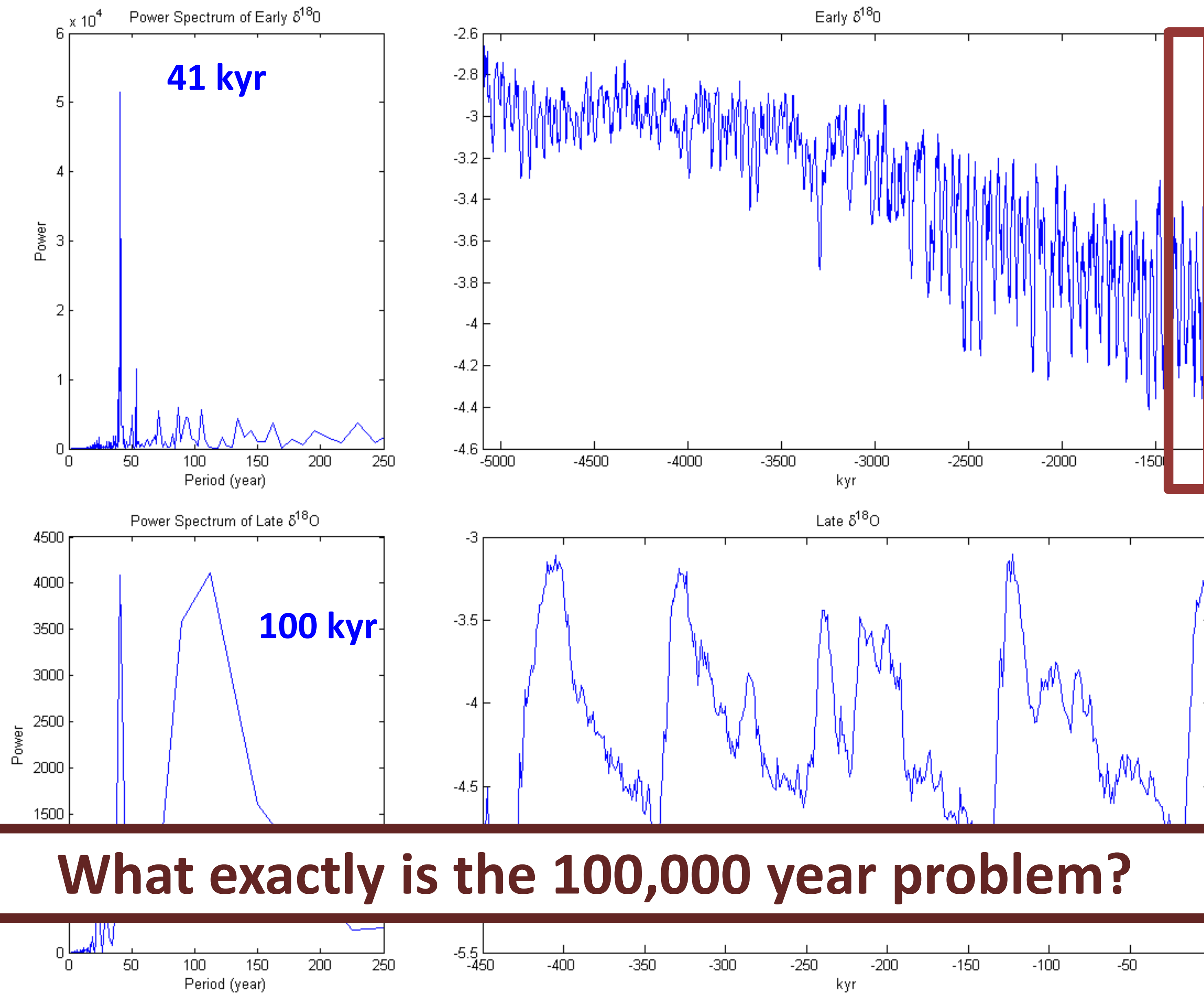
Introduction

mikecorthell.blogspot.com201106hot-earth-or-cold-earth-which-will-it.html



Introduction

Data from Lisiecki and Raymo, "A Pliocene-Pleistocene stack of 57 globally distributed benthic δ^{18}O records" *Paleoceanography* (2005), http://lorraine-lisiecki.com/LR04stack.txt.



What exactly is the 100,000 year problem?

Introduction

en.wikipedia.org/wiki/100,000-year_problem

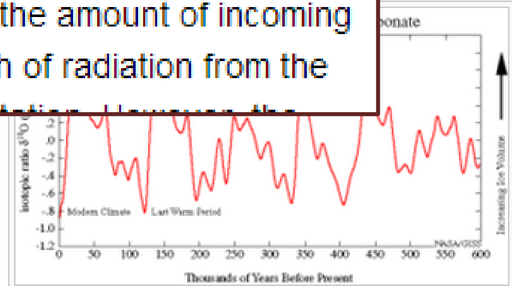
WIKIPEDIA
The Free Encyclopedia

100,000-year problem

From Wikipedia, the free encyclopedia

The **100,000 year problem** is a discrepancy between past temperatures and the amount of incoming solar radiation, or insolation. The latter rises and falls according to the strength of radiation from the

timescale, does not correlate well with these factors.

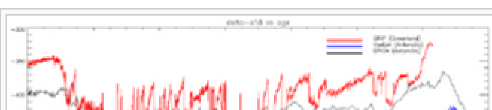


An average of several samples of $\delta^{18}O$, a proxy for temperature, for the last 600,000 years

Due to variations in the Earth's orbit, the amount of insolation varies with periods of around 21,000, 40,000, 100,000, and 400,000 years. Variations in the amount of incident solar energy drive changes in the climate of the Earth, and are recognised as a key factor in the timing of initiation and termination of glaciations. Spectral analysis shows the dominant periodicity of the climate response to be around 100,000 years, but the orbital forcing at this period is small.

Contents [hide]

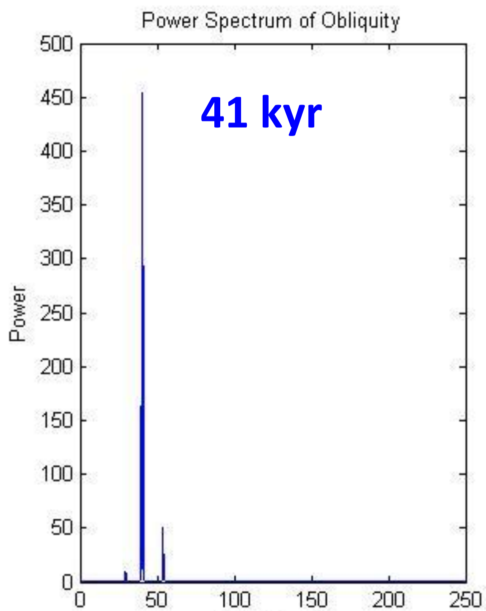
Reconstructing past climate [edit]

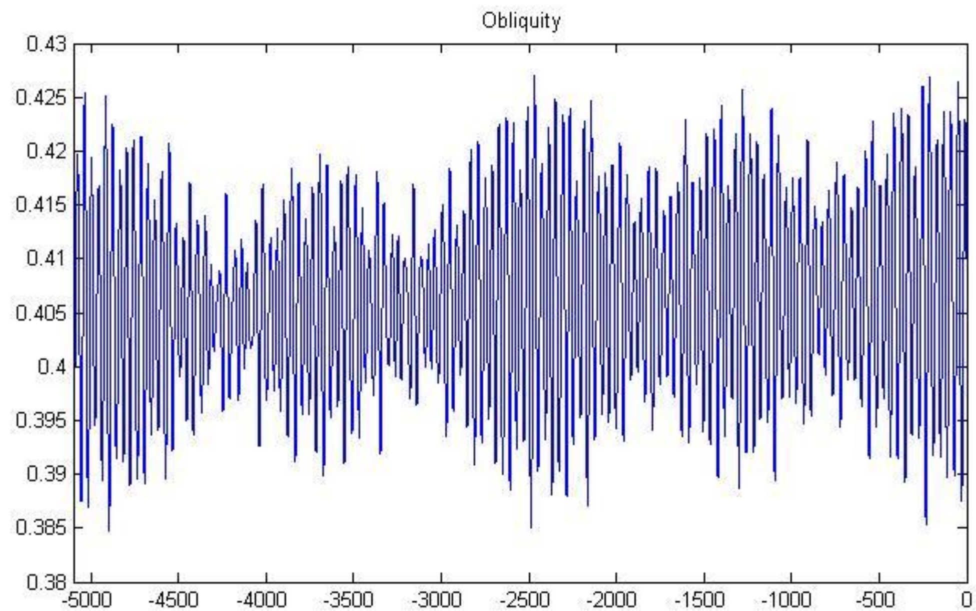


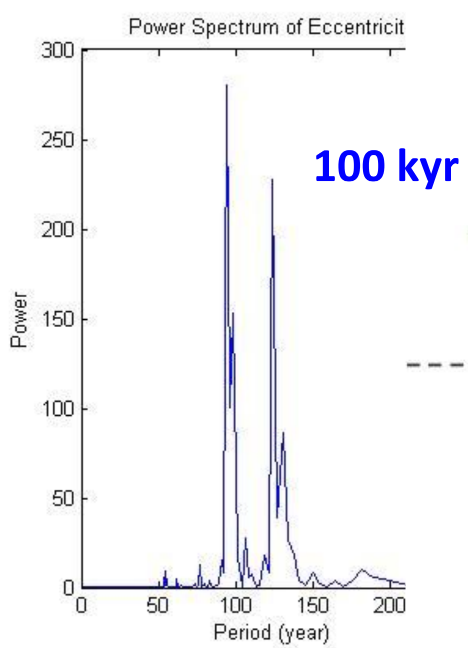
Past climate data—especially temperature—can be readily inferred from sedimentary evidence, although not with the accuracy that instruments can measure current temperatures. Perhaps the most useful indicator of past climate is the fractionation of oxygen isotopes, denoted $\delta^{18}O$. This fractionation

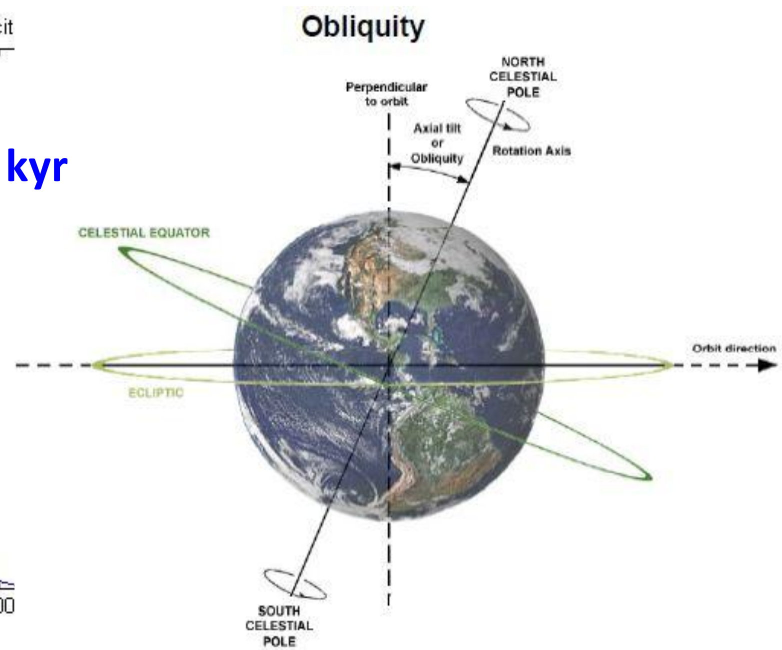
Introduction

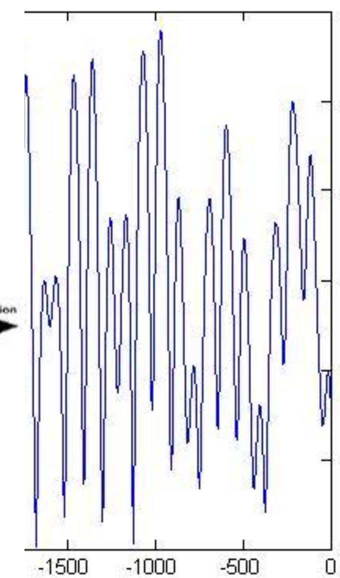
Data from Laskar et al, “A long-term numerical solution for the insolation quantities of the Earth” *Astronomy & Astrophysics* (2004), 261–285.
http://www.imcce.fr/Equipes/ASD/insola/earth/La2004/index.html



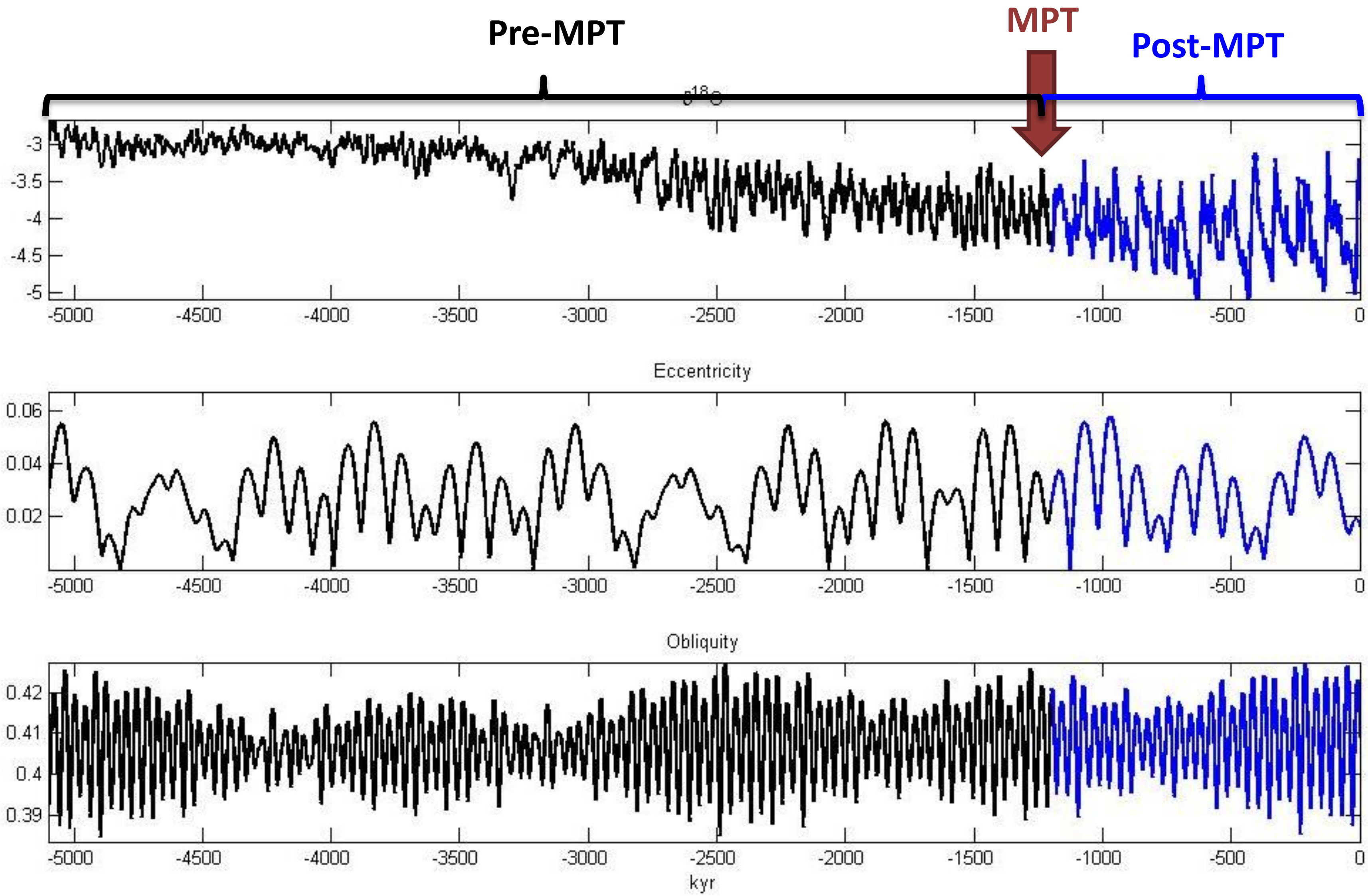








Introduction



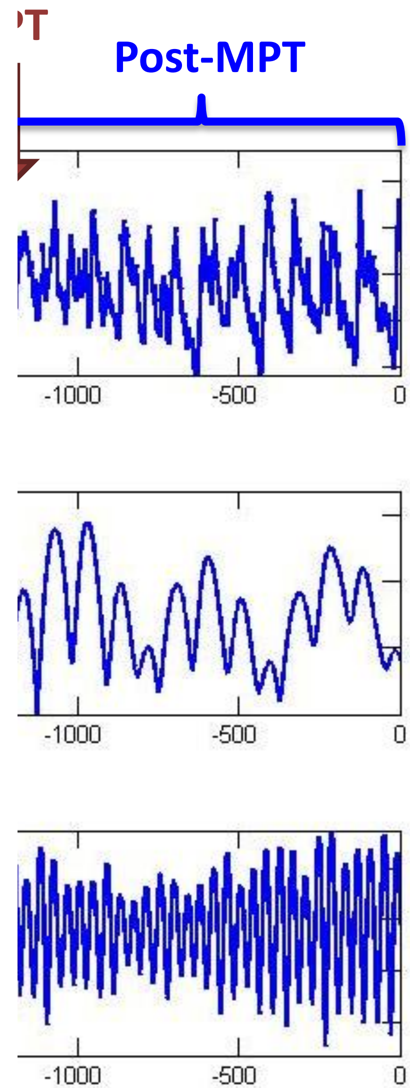
Introduction

Obliquity – 41kyr
Eccentricity – 100kyr

Pre-MPT – 41 kyr oscillations
Post-MPT – 100 kyr oscillations

So Earth transferred from obliquity to eccentricity forcing right?

WRONG!

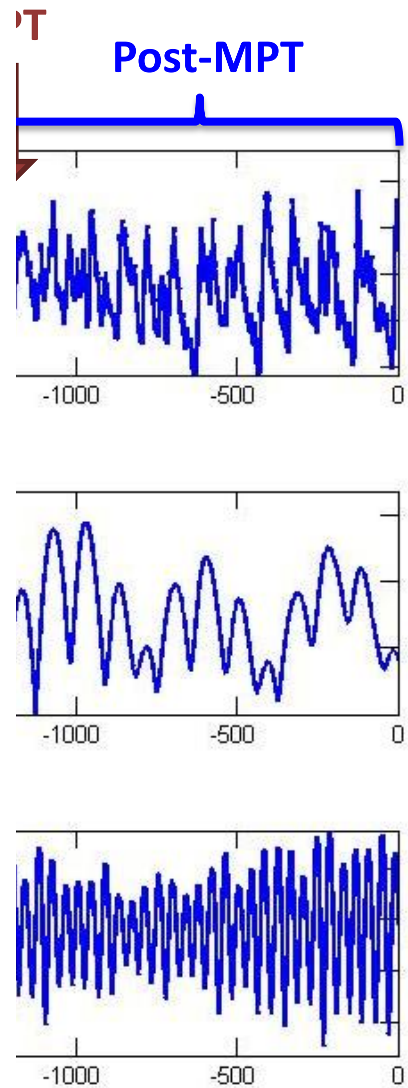


Introduction

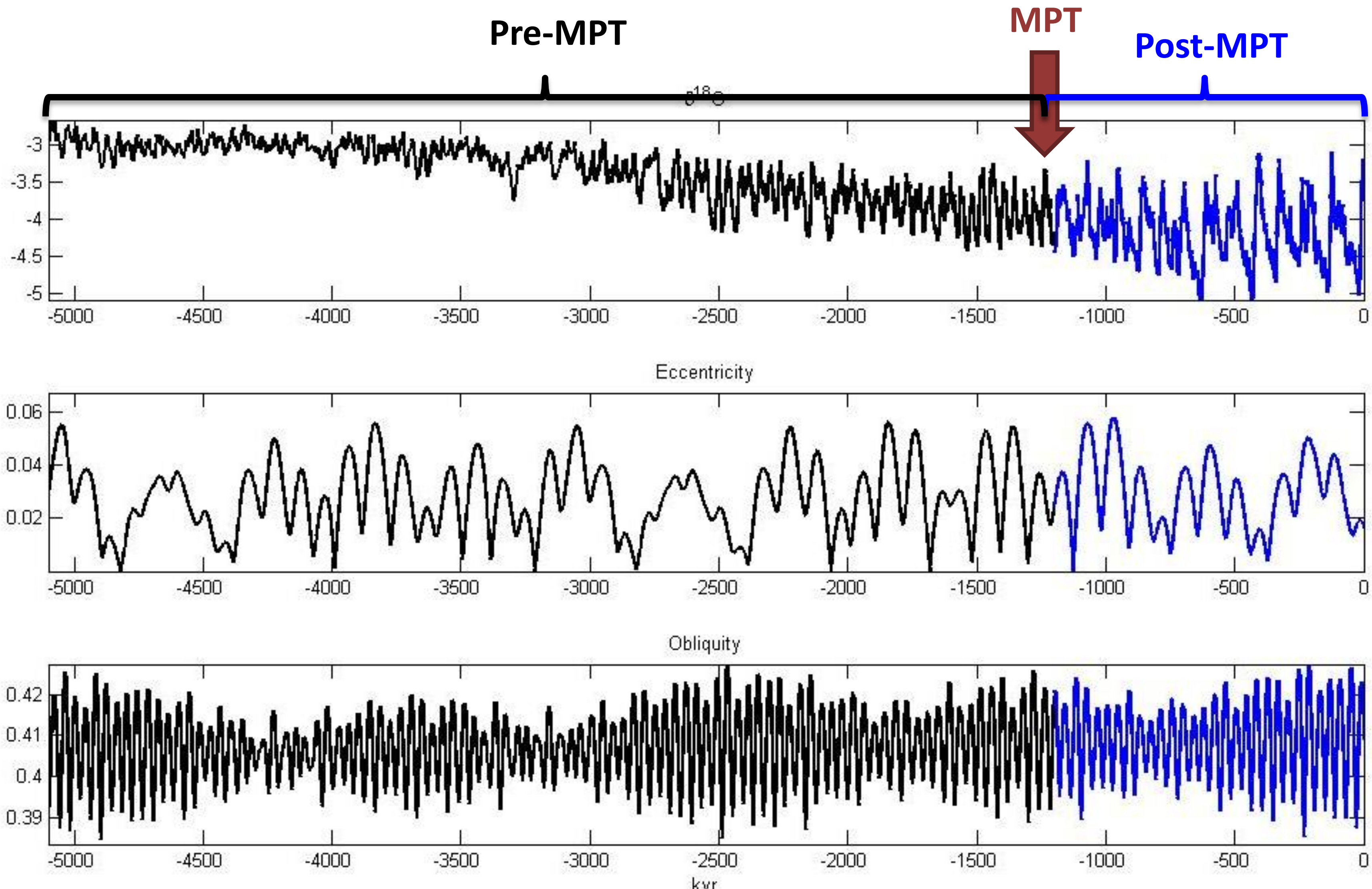
An air of mystery surrounds the MPT.

Although the new period matches that of eccentricity, the power of the eccentricity signal is much smaller than that of obliquity and is decreasing.

There is a nonlinear relationship between the Milankovtich cycles and δ^{18}O.

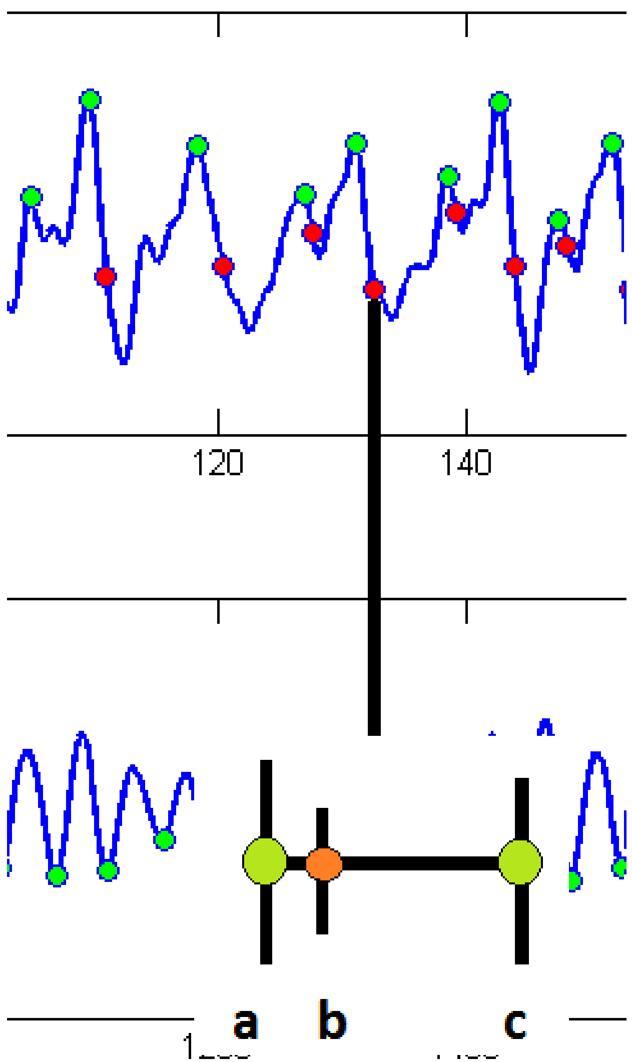


Introduction

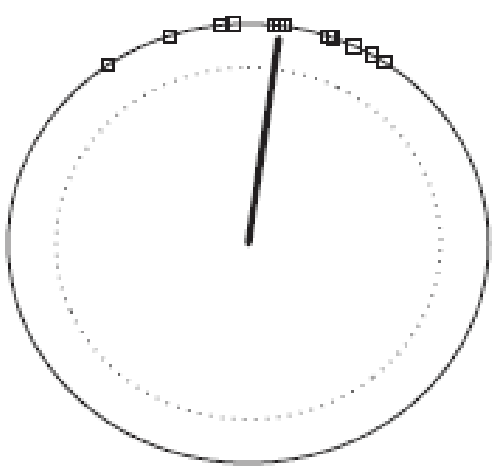


What do we know about the relationship between δ^{18}O and external forcing?

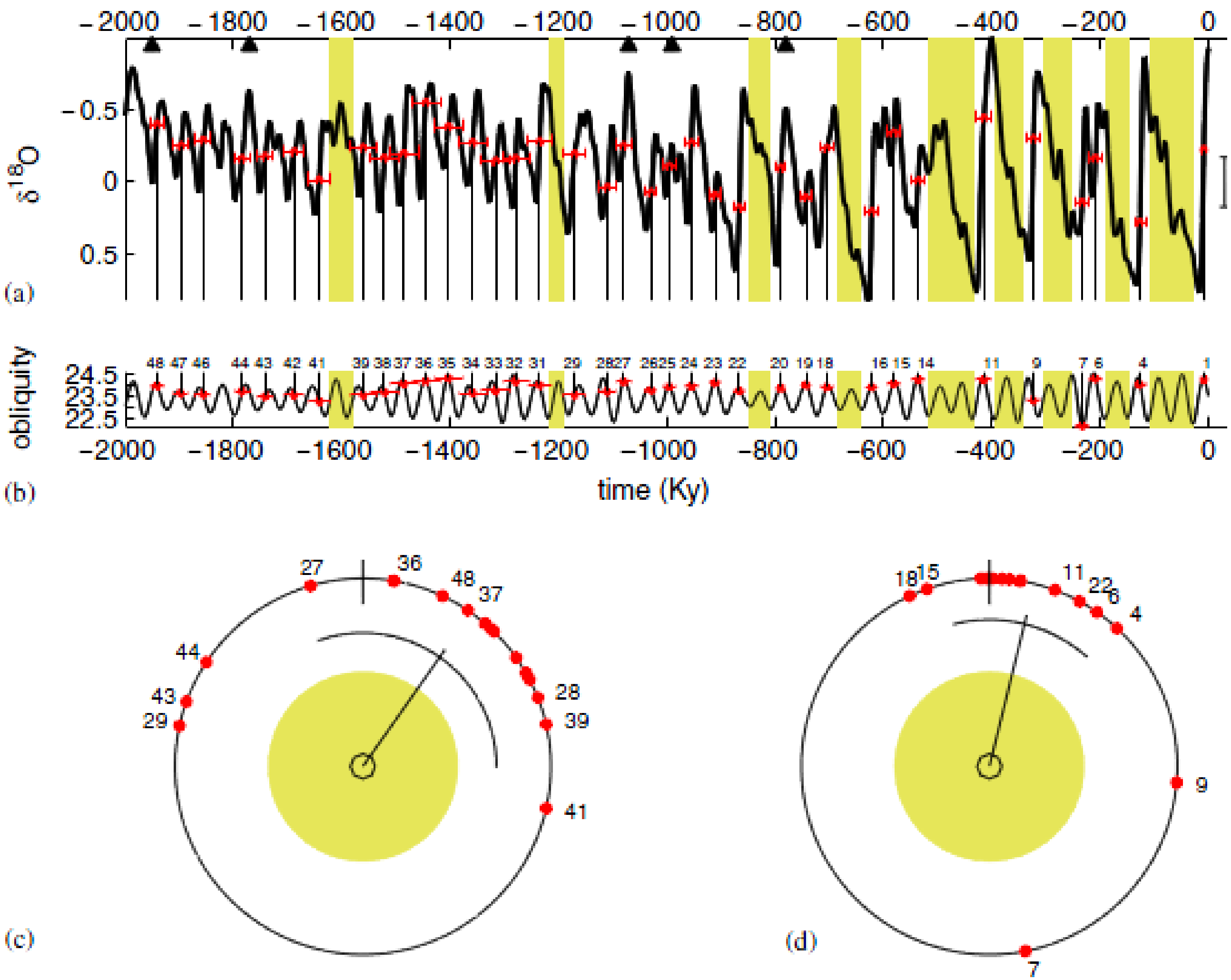
Computing Phase Angles



$$\Delta\text{Phase} = \left(\frac{b-a}{c-a}\right)2\pi$$

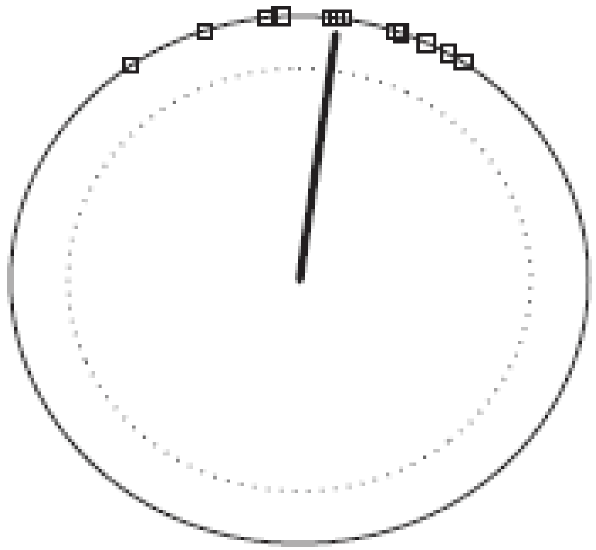


Phase Angles in δ^{18}O



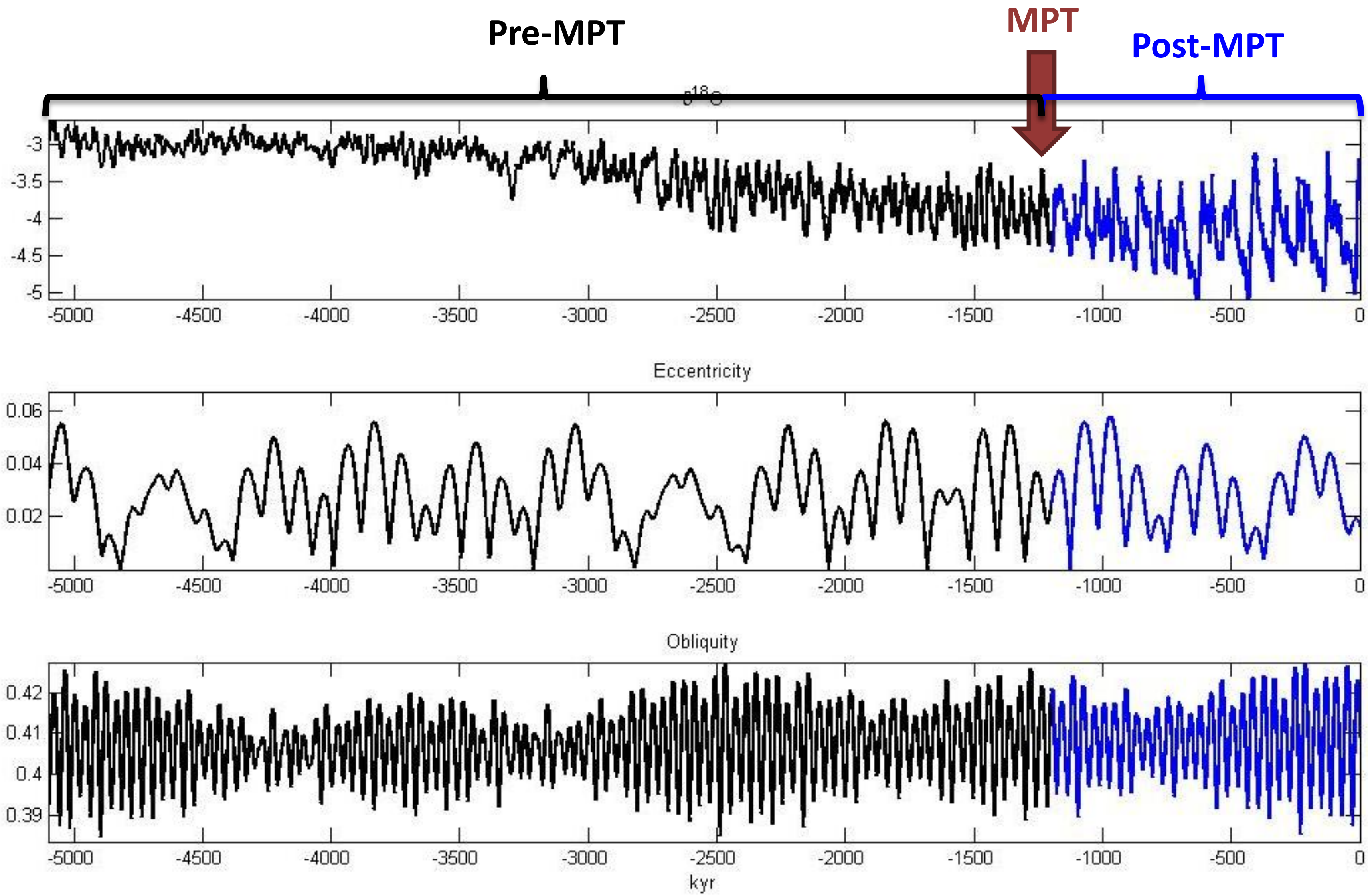
"During the early Pleistocene deglaciations occur nearly every obliquity cycle giving a 40 Ka timescale, while late Pleistocene deglaciations more often skip one or two obliquity beats, corresponding to 80 or 120 Ka glacial cycles which, on average, give the 100 Ka variability." [Huybers 2007, 2011]

Phase Angles in $\delta^{18}O$



"The relative phase of eccentricity and glacial cycles has been stable since 1.2 Myr ago, supporting the hypothesis that 100,000 glacial cycles are paced by eccentricity." [Lisiecki 2010]

Phase Angles in $\delta^{18}O$



$\delta^{18}O$ is in phase with obliquity and eccentricity for the last 2 mya and 1.2 mya. resp.

Table of Models

Identifying which type of model is the best choice for modeling the MPT is just as important as the actual fit of the model. The underlying mathematical structure might teach us something about the underlying drivers of the system.

MPT modeling options:

1. Dynamic Hopf Bifurcations
2. Relaxation Oscillators
3. Threshold/bursting models
4. Excitable System with Slow Manifold

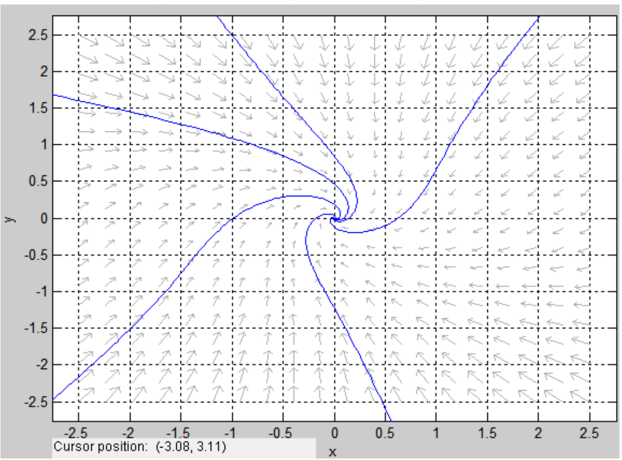
We will focus on the Dynamic Hopf Bifurcation as possible tool to uncover the secrets of the 100,000 year Problem. We will not consider methods 2-4.[1]

[1]To learn more about these systems I recommend Michel Crucifix's "Oscillators and relaxation phenomena in Pleistocene climate theory" 2012.

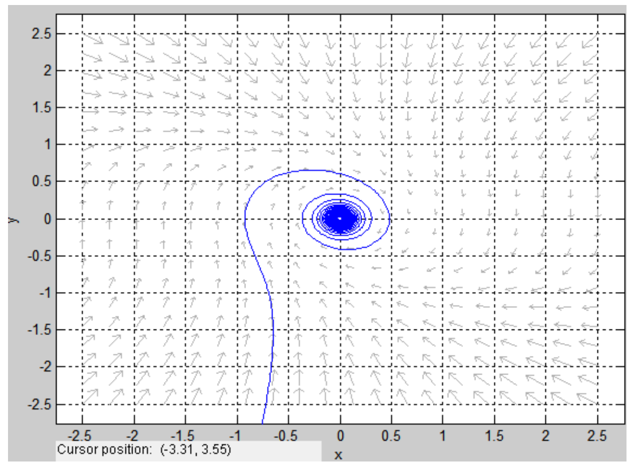
Hopf Bifurcation

$$\dot{x} = y + \mu x - xy^2$$
$$\dot{y} = \mu y - x - y^3$$

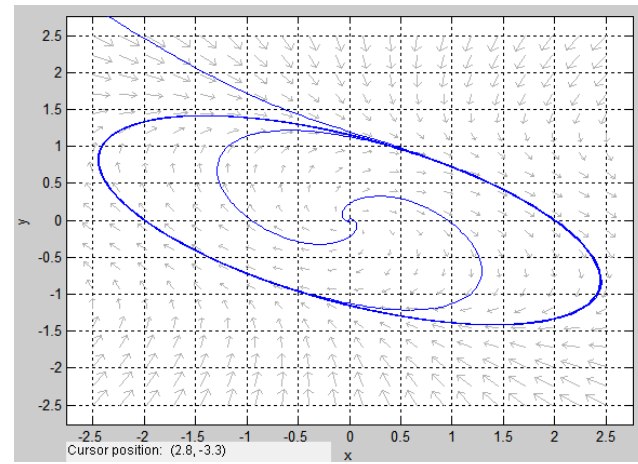
Velocity field/phase portrait:



$\mu < 0$



$\mu = 0$



$\mu > 0$

origin is a center.

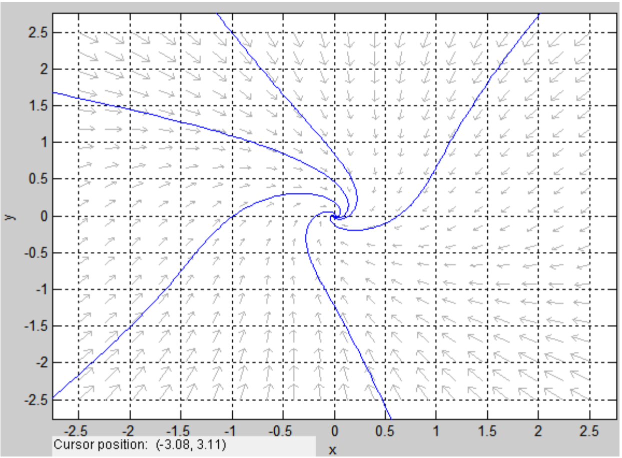
origin is globally stable or a global attractor

origin is locally unstable with a globally stable limit cycle

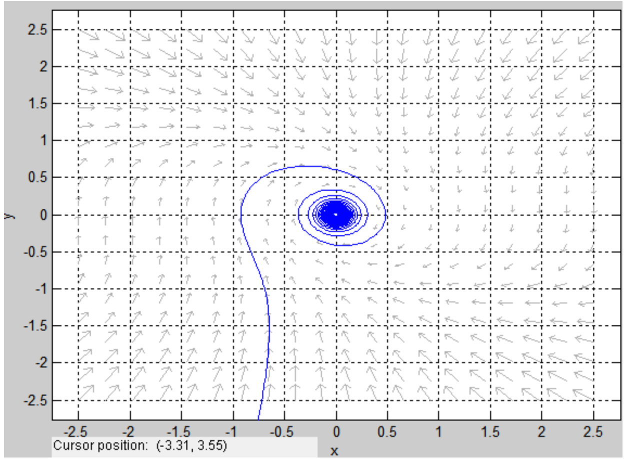
Dynamic Hopf Bifurcation

$$\dot{x} = y + \mu x - x y^2$$
$$\dot{y} = \mathrm{t}\, y - x - y^3$$

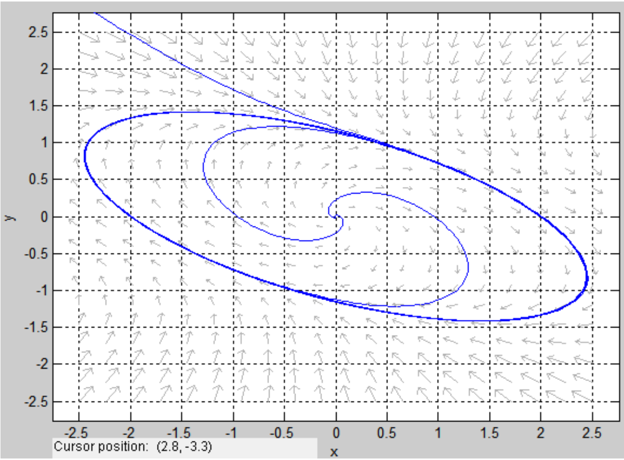
Velocity field/phase portrait:



$\mathrm{t} < 0$



$\mathrm{t} = 0$



$\mathrm{t} > 0$

origin is a center.

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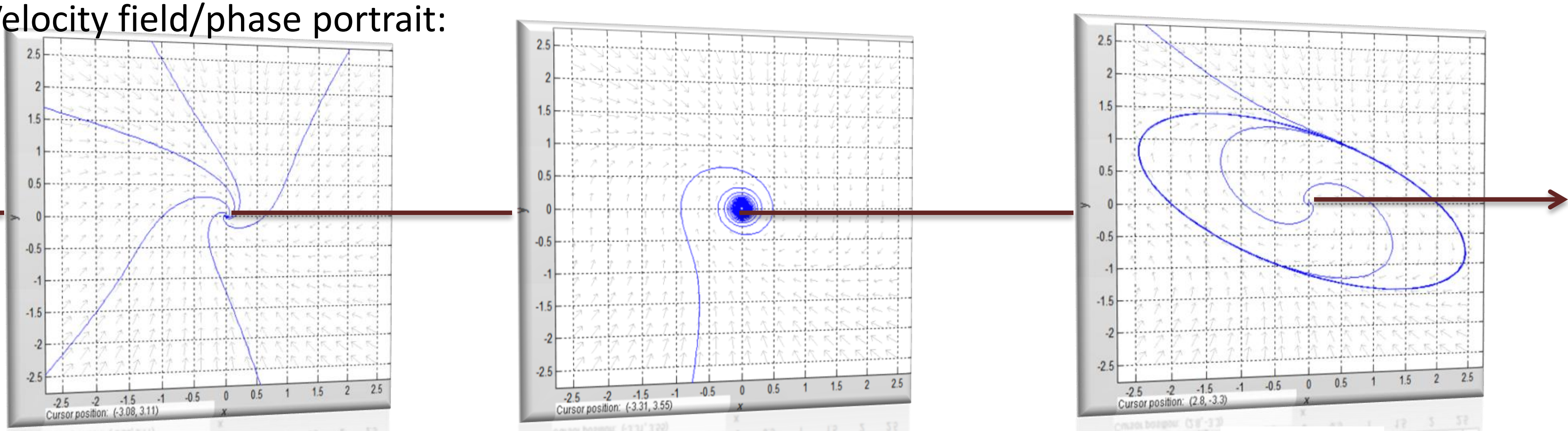
Dynamic Hopf Bifurcation

$$\dot{x} = y + \mu x - xy^2$$
$$\dot{y} = t\,y - x - y^3$$

Velocity field/phase portrait:



$t < 0$ $t = 0$ $t > 0$

origin is a center.

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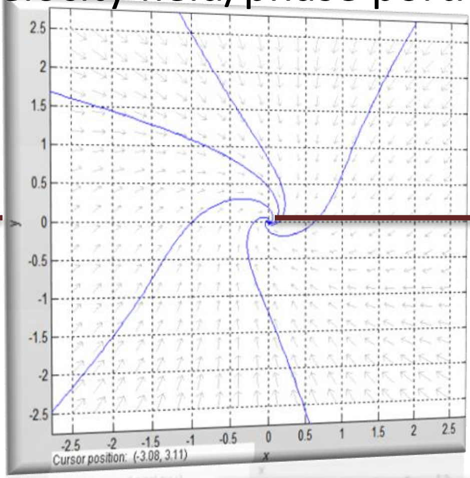

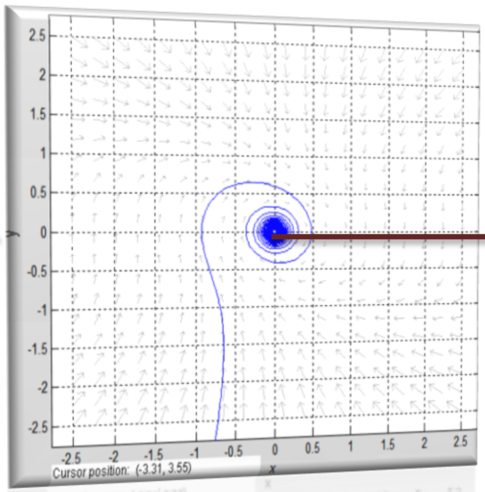

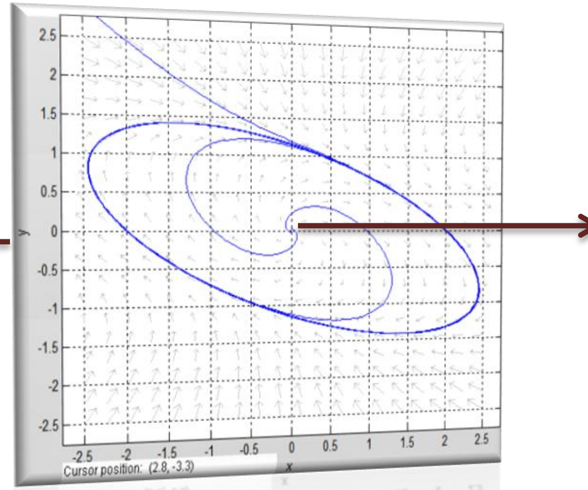

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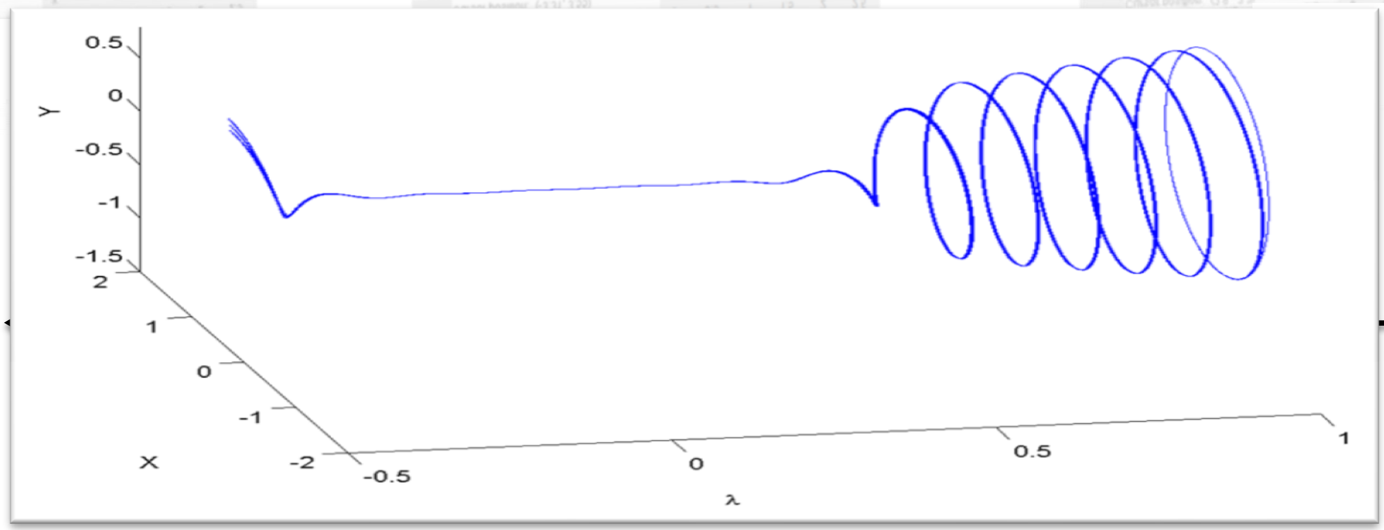


0

Dynamic Hopf Bifurcation

Because the δ^{18}O data has small oscillations followed by larger ones, it is reasonable to model the MPT with a forced dynamic Hopf Bifurcation.

Two famous examples are all the Saltzman Models (1987, 1988, 1990, 2001) and Korobeinikov (2010).

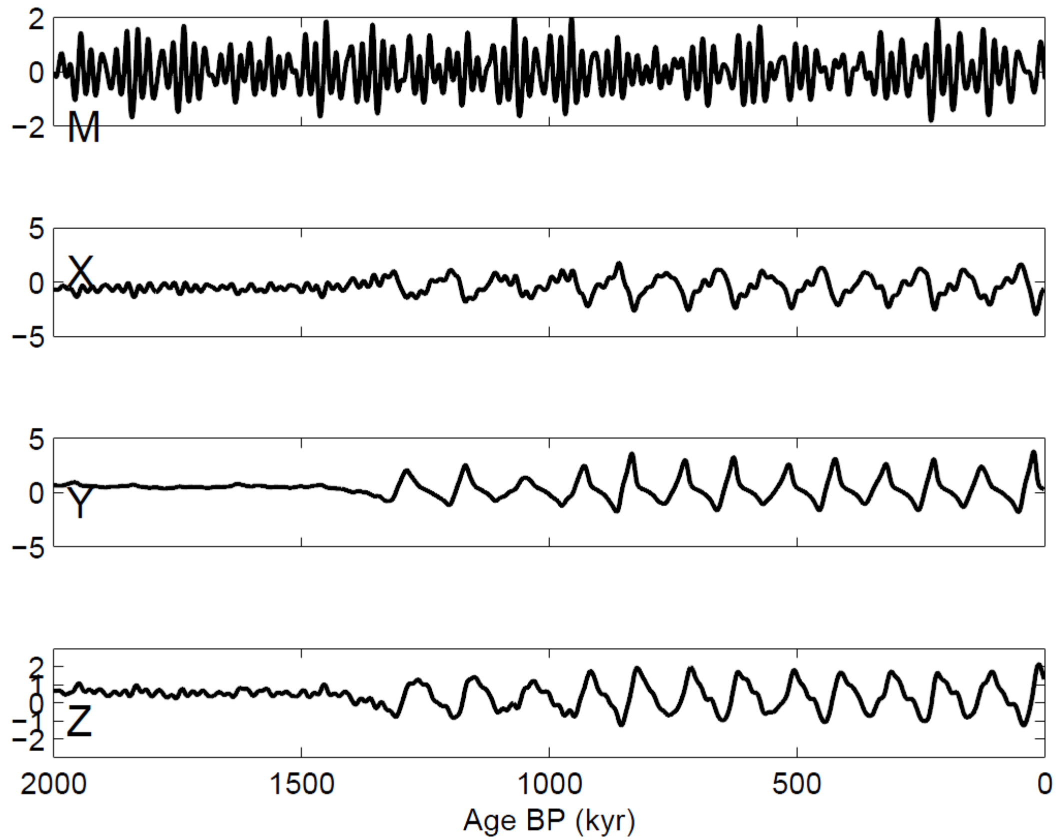
We explore one of most famous Dynamic Hopf Bifurcation MPT models from Barry Saltzman and Kurt Maasch.

Maasch & Saltzman [1990]

Ice Line $\dot{X} = -X - Y - uM(t)$

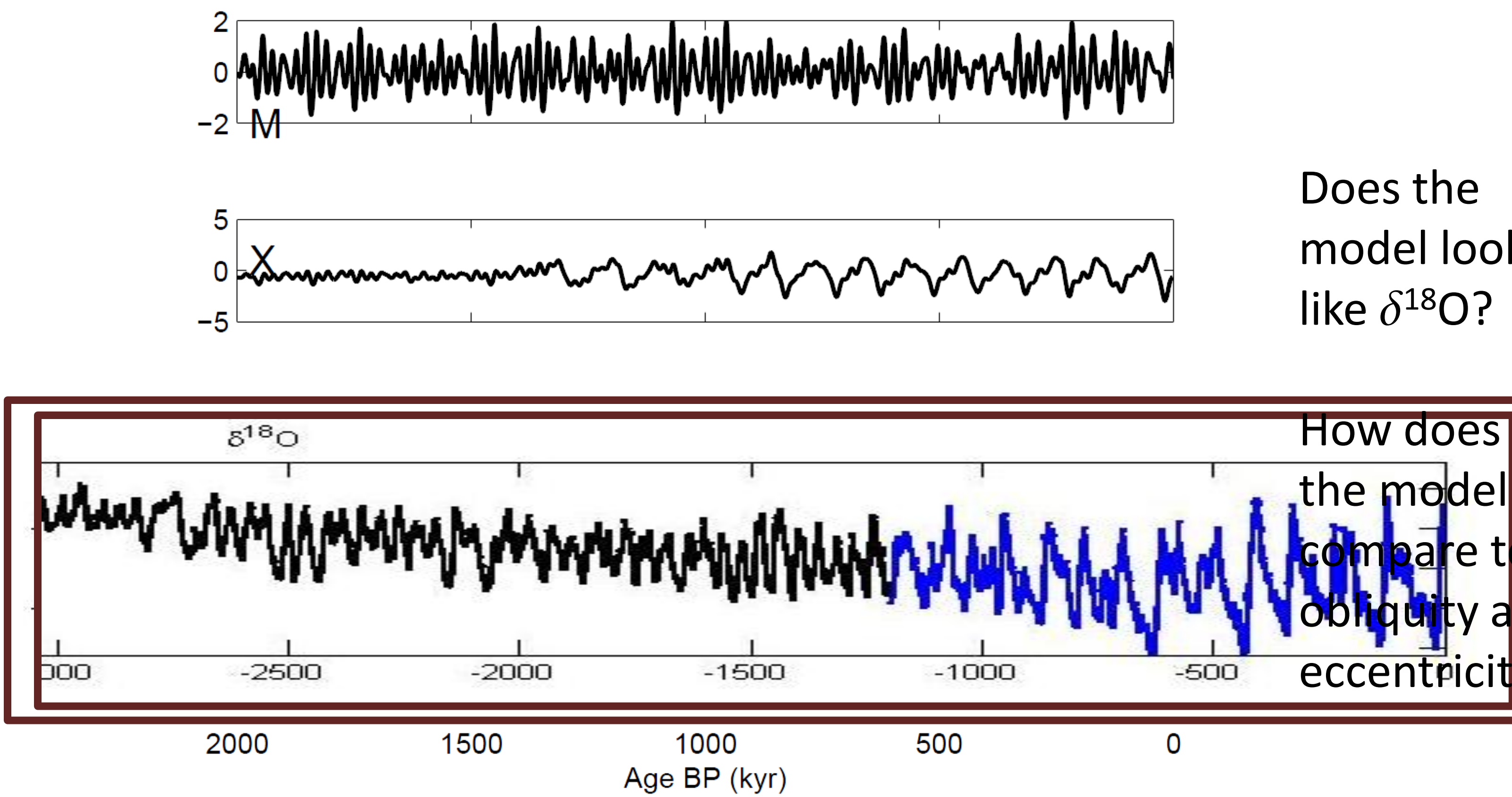
Atmospheric CO_2 $\dot{Y} = -pZ + rY + sZ^2 - Z^2Y$

North Atlantic Deep Water Formation $\dot{Z} = -q(X + Z).$



Maasch & Saltzman [1990]

Ice Line	$\dot{X}$	$=$	$-X - Y - uM(t)$
Atmospheric CO_2	$\dot{Y}$	$=$	$-pZ + rY + sZ^2 - Z^2Y$
North Atlantic Deep Water Formation	$\dot{Z}$	$=$	$-q(X + Z).$

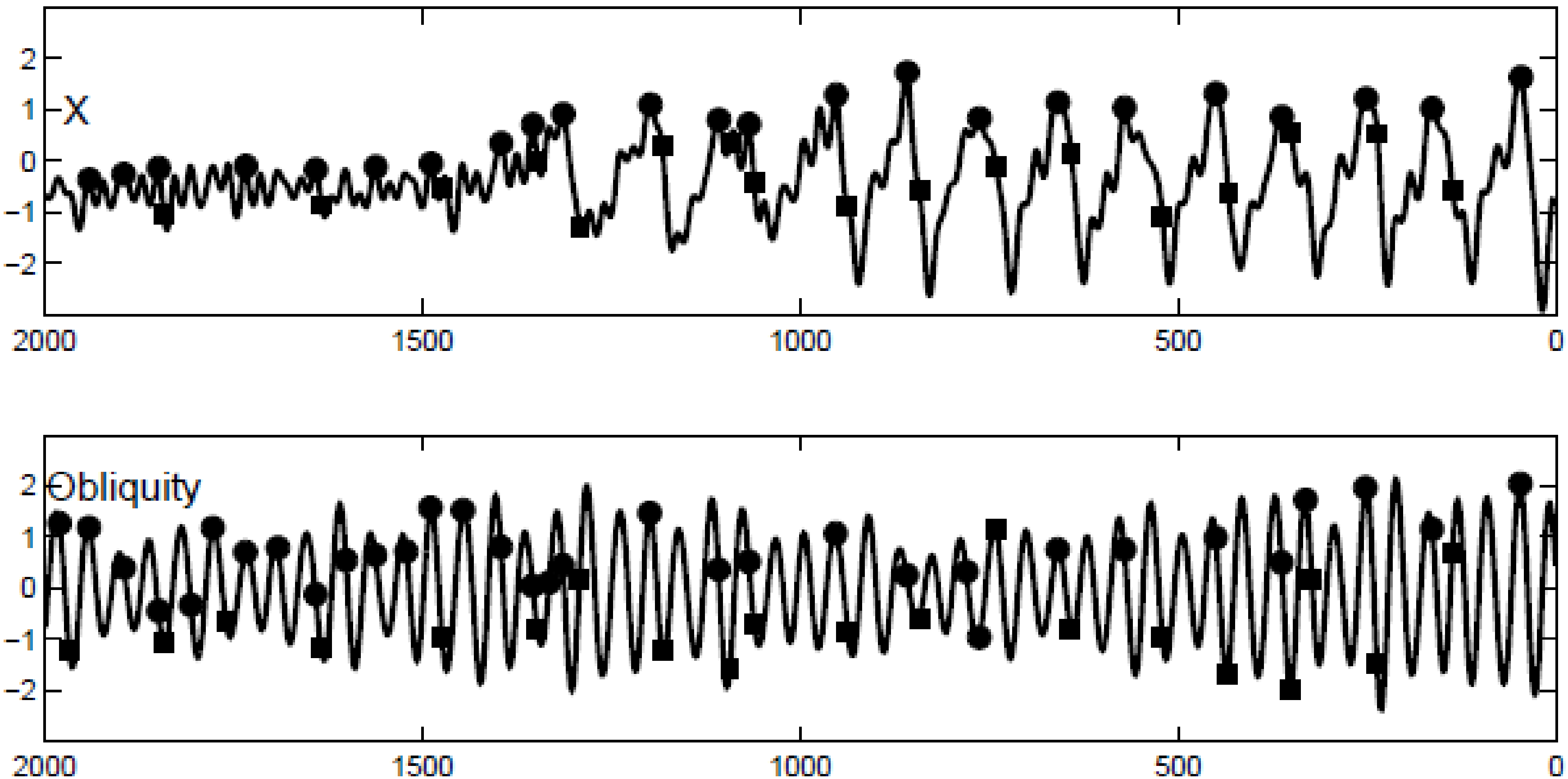


Does the model look like δ^{18}O?

How does the model compare to obliquity and eccentricity?

Dynamic Hopf Bifurcation

How does the model compare to obliquity?



Maasch & Saltzman [1990]

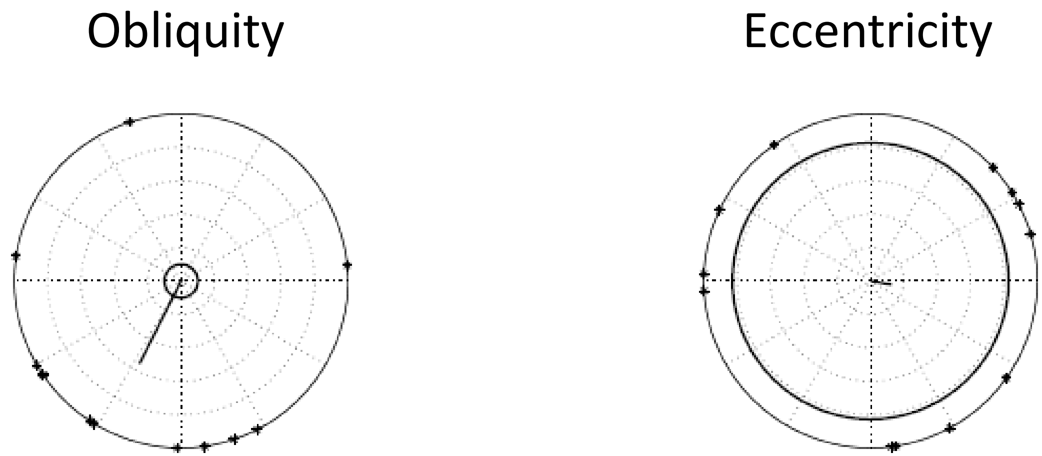
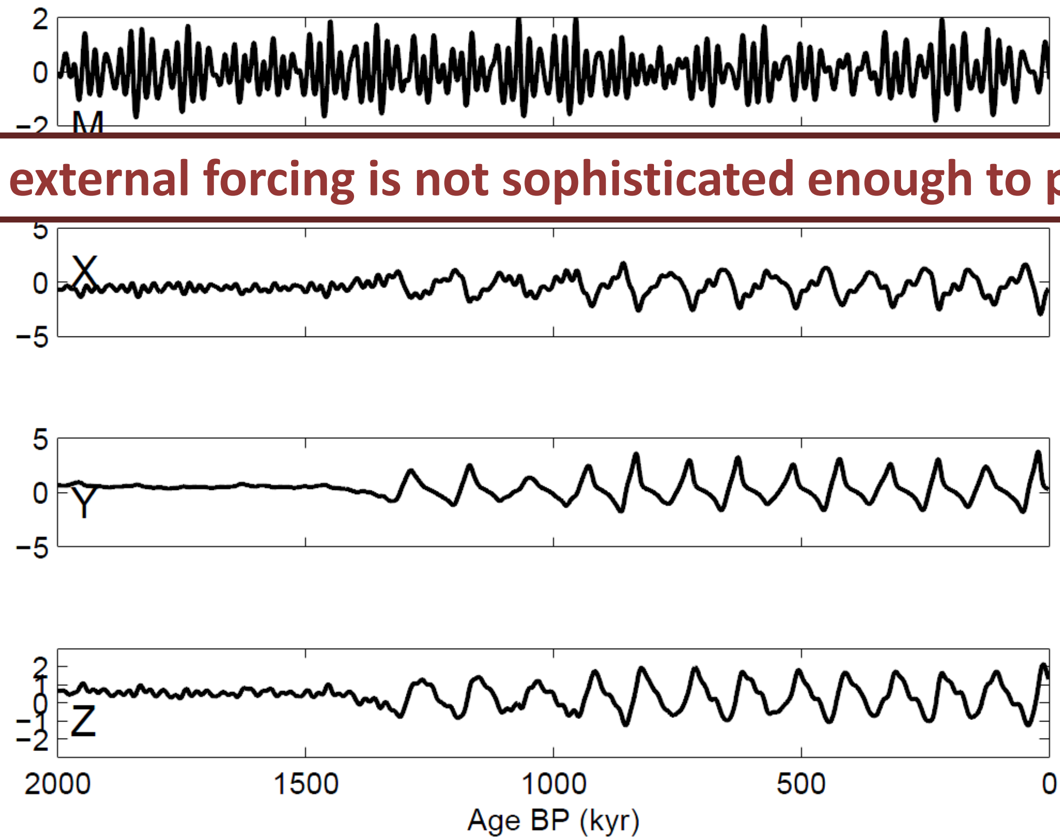


Figure 3.2: Circular statistics of the last 1.2 million years. Obliquity phase angle is on the left, eccentricity is on the right. Radial line shows mean angle with magnitude R showing relative cohesion of angles. The inner solid circle is the magnitude R must be exceed to reject H_0. Stars along the outer circle are individual phase angle differences. A radial line pointing straight up would show the model is in phase with the local maxima of the forcing term. A radial line pointing straight down would show the model is in phase with the local minima of the forcing term.

Dynamic Hopf Bifurcation

Ice Line	$\dot{X} = -X - Y - uM(t)$
Atmospheric CO_2	$\dot{Y} = -pZ + rY + sZ^2 - Z^2Y$
North Atlantic Deep Water Formation	$\dot{Z} = -q(X + Z).$



Maybe the external forcing is not sophisticated enough to phase correlate?

Improving Maasch and Saltzman

Budyko-Sellers-Widiasih Model:

$$\frac{\partial T}{\partial t} = \frac{k}{R}((1-\alpha(y,\eta))Qs(y) - (A + BT(y)) + C(\overline{T} - T(y))$$

$$\frac{d\eta}{dt} = k\epsilon(T(\eta) - T_c)$$

With attracting invariant curve found by Widiasih-McGehee:

$$\dot{\eta} = \epsilon h(\eta)$$

$$h(\eta) = \left(\Phi_0(\eta) + \frac{Qs_2(1-\alpha_0)}{B+C}p_2(\eta) - T_c\right)$$

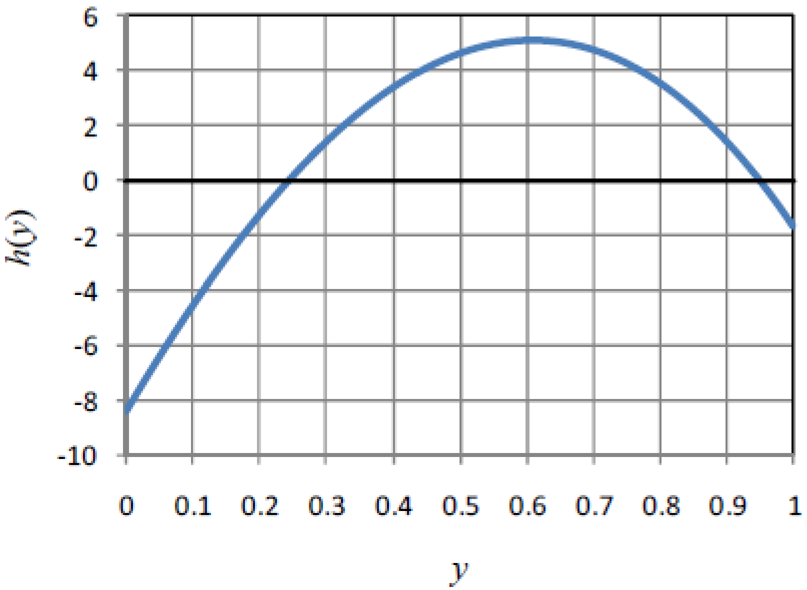
$$\Phi_0(\eta) = \frac{1}{B}\left(Q(1-\alpha_0) - A + C\frac{Q(\alpha_2-\alpha_1)}{B+C}\left(\eta - \frac{1}{2} + s_2P_2(\eta)\right)\right)$$

where

$$s_2(\beta) = \frac{5}{16}\left(-2 + 3\sin^2\beta\right)$$

$$p_2(y) = \tfrac{1}{2}(3y^2 - 1)$$

$$P_2 = \int_0^\eta p_2(y)dy = \frac{1}{2}(\eta^3 - \eta)$$



Improving Maasch and Saltzman

We incorporate the Budyko-Sellers-Widiasih Model into the Maasch & Saltzman model:

$$\begin{aligned}
\dot{\eta} &= \epsilon\left(\left(\frac{C(\omega)Q(\varepsilon)(\alpha_2-\alpha_1)}{B(B+C(\omega))}\right)\left(\eta-\frac{s_2(\beta)\eta}{2}\right)+\frac{Q(\varepsilon)}{B}(1-\alpha_0)\right)- \\
&\quad \left(\frac{A(\mu)}{B}+\left(\frac{1-s_2(\beta)\eta^3}{2}\right)\frac{C(\omega)Q(\varepsilon)(\alpha_2-\alpha_1)}{B(B+C(\omega))}\right)+\left(\frac{Q(\varepsilon)s_2(\beta)(1-\alpha_0)}{2(B+C(\omega))}(3\eta^2-1)\right)-T_c \\
\dot{\mu} &= -p\omega+r\mu+s\omega^2-m\omega^2\mu \qquad (4.1) \\
\dot{\omega} &= -q(f(\eta)+\omega)
\end{aligned}$$

$$\begin{aligned}
\alpha_1 &= 0.32 & p &= 0.8 & A(\mu) &= -3\mu+205.5 \\
\alpha_2 &= 0.62 & q &= 1.8 & C(\omega) &= 0.05\omega+3 \\
\alpha_0 &= (\alpha_1+\alpha_2)/2 & m &= 1 & f(\eta) &= -16\eta+13.2 \\
B &= 1.9 & T_c &= -10 & s_2 &= (5/16)(-2+3\sin\beta^2)
\end{aligned}$$

Improving Maasch and Saltzman

We incorporate the Budyko-Sellers-Widiasih Model into the Maasch & Saltzman model:

$$
\begin{aligned}
\dot{\eta} &= \epsilon\left(\left(\frac{C(\omega)Q(\varepsilon)(\alpha_2-\alpha_1)}{B(B+C(\omega))}\right)\left(\eta-\frac{s_2(\beta)\eta}{2}\right)+\frac{Q(\varepsilon)}{B}(1-\alpha_0)\right)- \\
&\quad \left(\frac{A(\mu)}{B}+\left(\frac{1-s_2(\beta)\eta^3}{2}\right)\frac{C(\omega)Q(\varepsilon)(\alpha_2-\alpha_1)}{B(B+C(\omega))}\right)+\left(\frac{Q(\varepsilon)s_2(\beta)(1-\alpha_0)}{2(B+C(\omega))}(3\eta^2-1)\right)-T_c \\
\dot{\mu} &= -p\omega+r\mu+s\omega^2-m\omega^2\mu \qquad (4.1) \\
\dot{\omega} &= -q(f(\eta)+\omega)
\end{aligned}
$$

$$
\begin{aligned}
\alpha_1 &= 0.32 & p &= 0.8 \\
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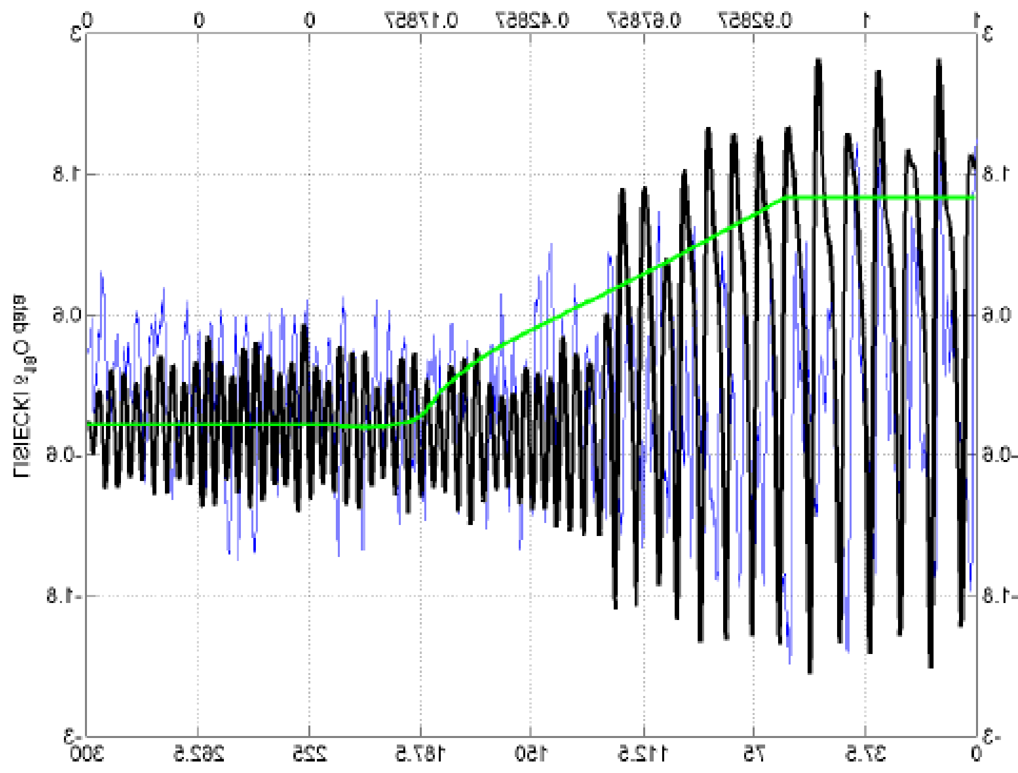
B &= 1.9 & T_c &= -10
\end{aligned}
$$



Improving Maasch and Saltzman

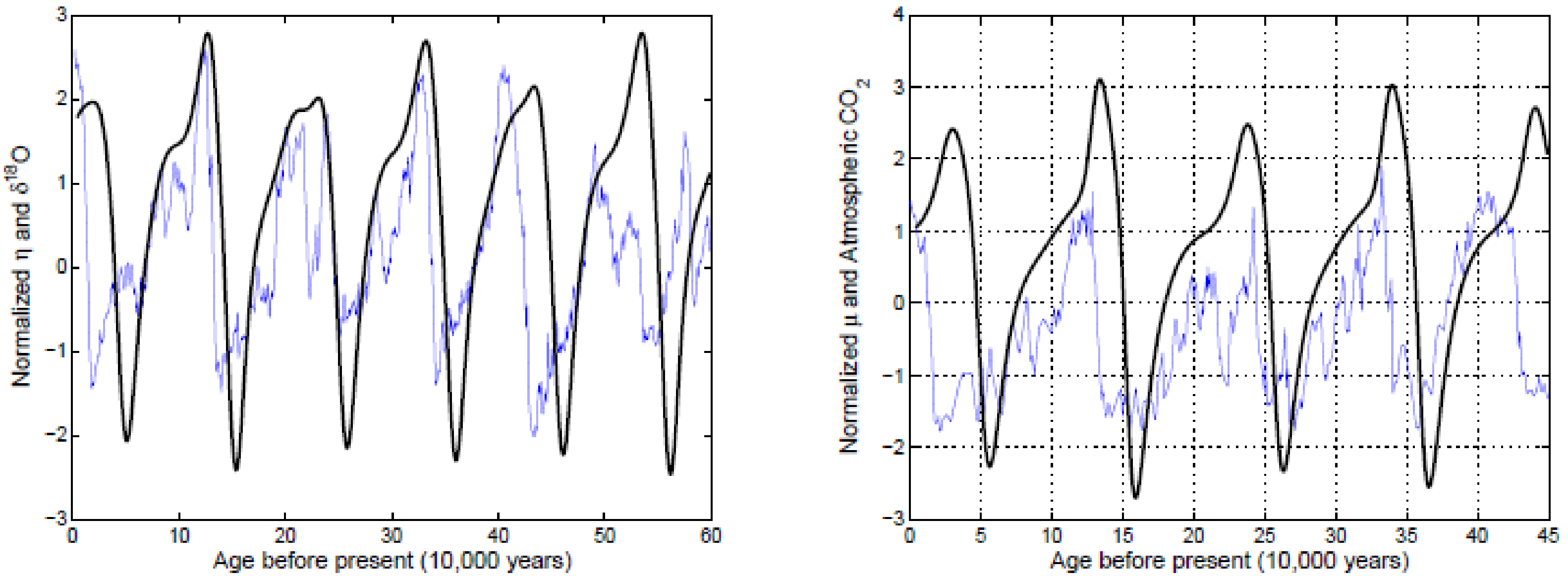


Figure 4.1: Left: Ice mass data for the last 3My. Blue data is δ^{18}O data from the supplement to Lisiecki (2005), Black curve is output from the model in Equation 4.1 [34]. Right: Atmospheric CO_2 comparison for the last 450 kyrs. Normalized atmospheric CO_2 data from Luthi (2008) is shown in black, and normalized model output of atmospheric CO_2 is shown in blue [35].

Dynamic Hopf Bifurcation

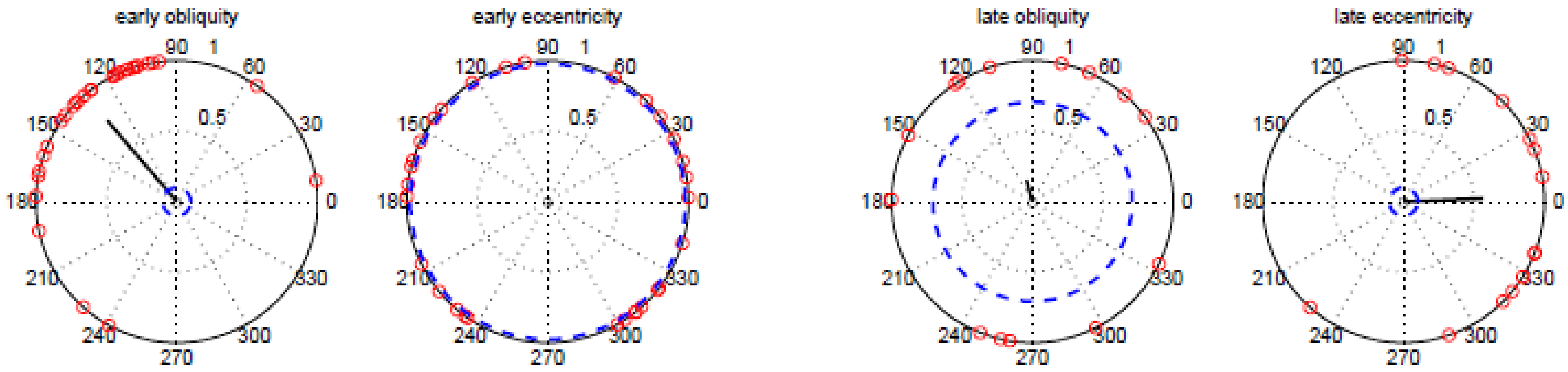
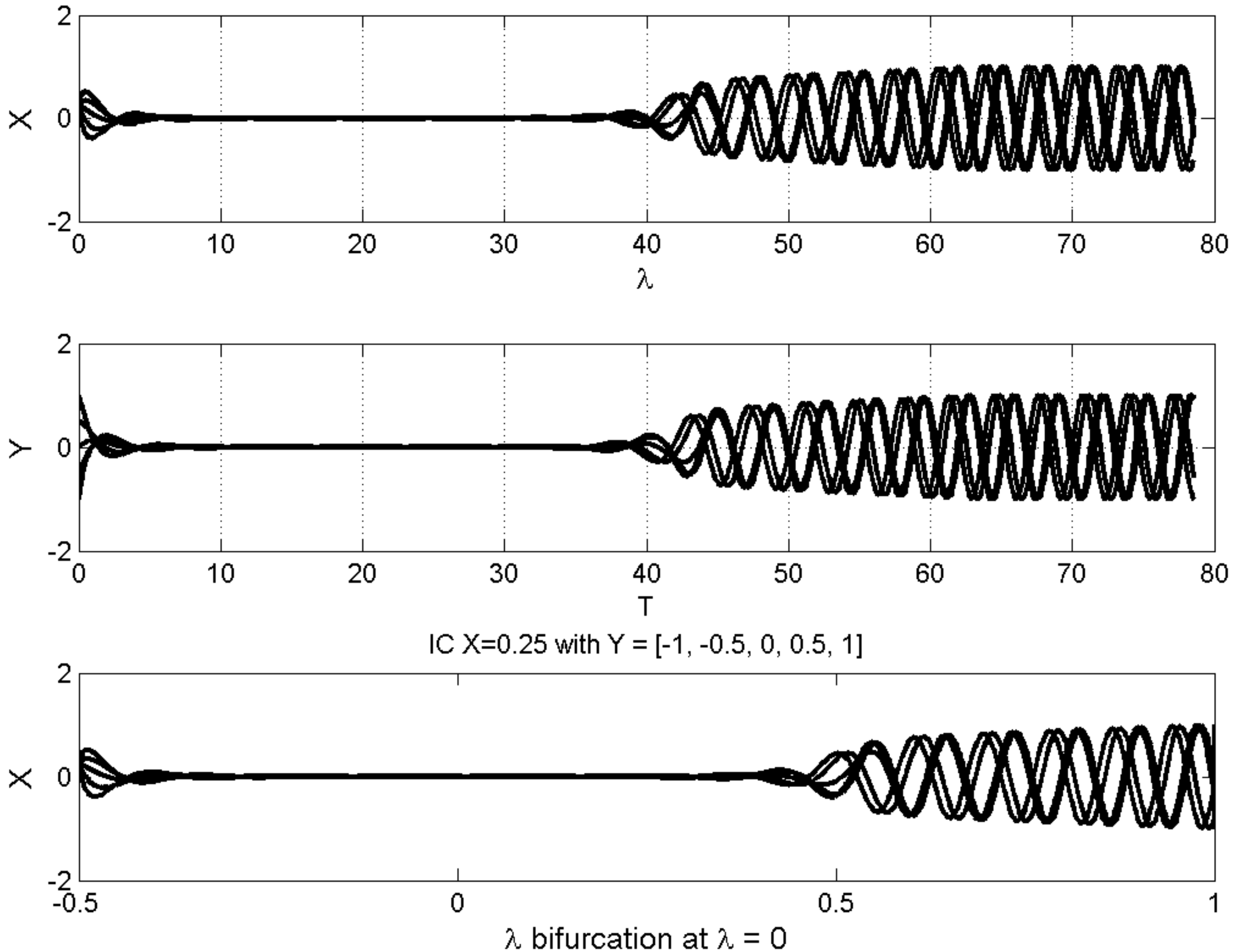


Figure 4.3: Phase angle analysis between model ice line output and orbital forcing for early (3 My - 1.2 My) and late (1.2 My to present) Pleistocene. The reader is referenced to Zar (1999) and Upton and Fingleton (1989) for details on the circular statistics used to produce these diagrams [31, 32]. Lisiecki (2010) also presents a concise review of the process [8].

Dynamic Hopf Bifurcation

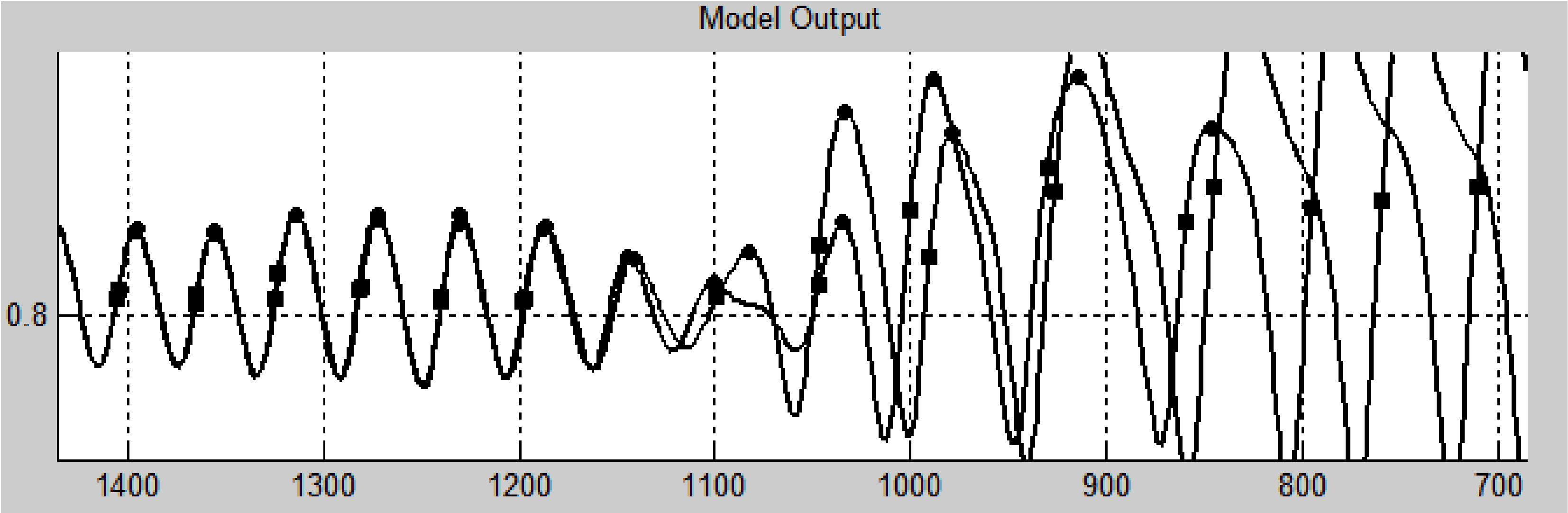
And it gets worse . . .

Initial Conditions of Dynamic Hopf



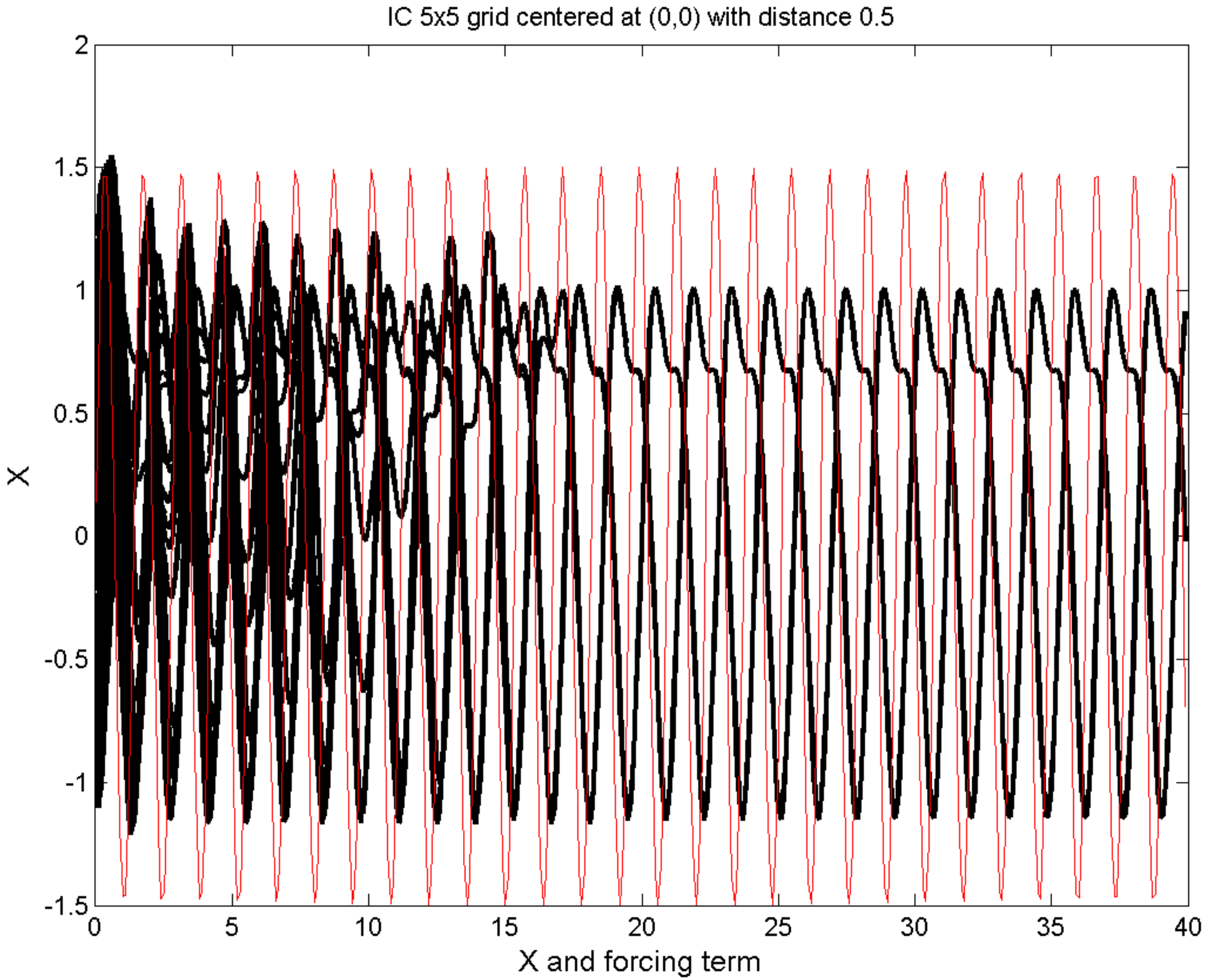
Sensitivity to Initial Conditions

Initial Conditions of Dynamic Hopf



One example of Sensitivity to Initial Condition at the neck.

Initial Conditions of Dynamic Hopf



Order in Chaos: phenomena of stable trajectories in a dynamical system.

Dynamic Hopf Bifurcation

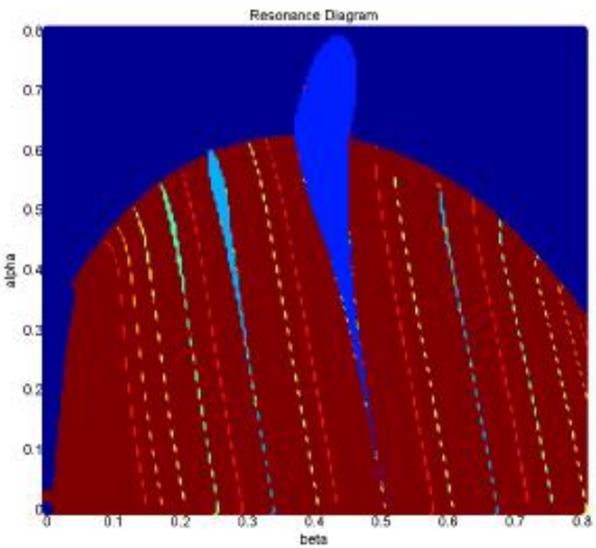
The Improved Maasch and Saltzman model suggests that an accurate accounting for Milankovitch cycles does not solve the phase angle discrepancy nor does it reduce or eliminate the initial condition sensitivity.

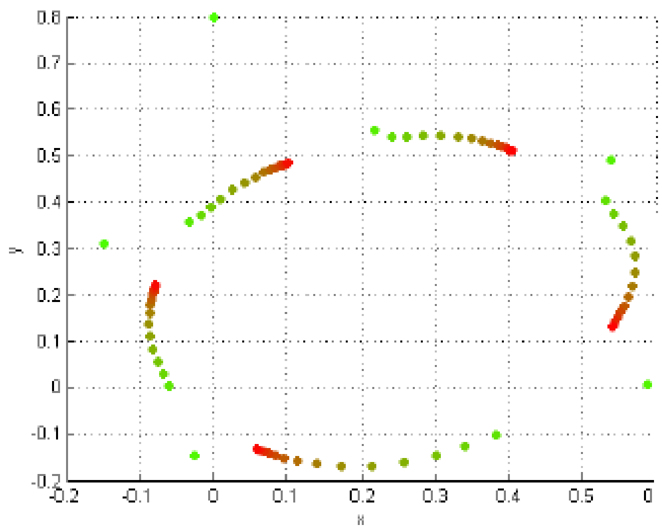
In order to formally question the assumption of phase angle correlation between a dynamic Hopf bifurcation and the external forcing, we must generalize the model…

Advertisement!

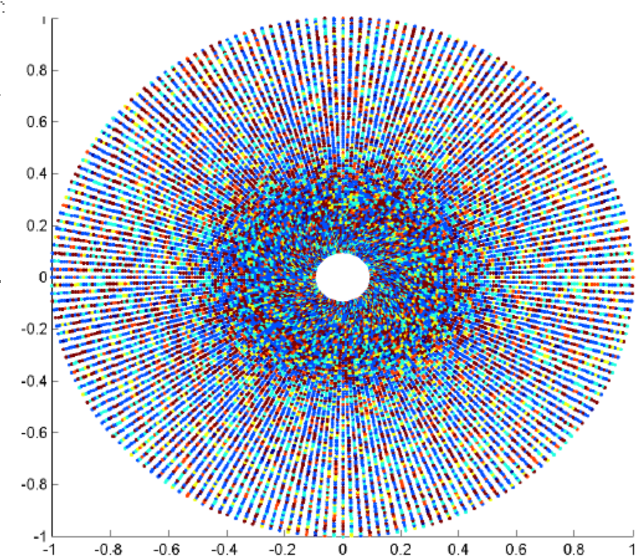
Tune in next week!

Poincare Section Maps!





Arnold Tongue Diagrams!



...And Colorful Dots!